TU Clausthal

Department of Informatics

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Exercise

Logic and Verification

– Sheet 1: Sentential Logic –

Date: 08. April 2019

Points:

______________ of 20

Group / Tutor:

Name(s) & Matr. no.:

Remarks: In order to be **permitted to the exam**,

- **at least 50%** of all exercise points have to be obtained,
- *and* on all but one exercise sheet at least **20%** of the points have to be obtained,
- *and* exercises have to be solved (and submitted) in groups of up to 2 students,
- *and* everybody needs to successfully present their solution to one of the exercises in the exercise groups **twice** during the semester.

Exercise 1 (10 Points, Validity, truth, and satisfiability)

Decide for each of the following sentences whether it is (1) a tautology, (2) satisfiable but no tautology, or (3) unsatisfiable, and prove it.

(a) $(\neg \mathsf{top} \rightarrow \neg \mathsf{small}) \rightarrow \neg \mathsf{top}$

(b) $(\neg\neg \mathsf{p} \rightarrow \neg\neg \mathsf{r}) \rightarrow (\mathsf{p} \rightarrow \mathsf{r})$

(c) $\neg \mathsf{red} \wedge (\mathsf{red} \vee \mathsf{green}) \wedge (\neg\neg \mathsf{green} \rightarrow \mathsf{red})$

(d) $\square \rightarrow (\top \wedge (\neg \mathsf{perfect} \vee \neg \mathsf{r} \leftrightarrow (\mathsf{q} \wedge \mathsf{s}))$

(e) $\mathsf{a} \rightarrow (\mathsf{b} \rightarrow (\mathsf{a} \rightarrow (\mathsf{b} \rightarrow (\mathsf{a} \rightarrow (\mathsf{b} \rightarrow (\mathsf{a} \rightarrow (\mathsf{b} \rightarrow (\mathsf{a} \rightarrow \mathsf{b}))))))))$

(f) $(\mathsf{a} \vee \mathsf{b}) \wedge (\mathsf{c} \vee (\neg \mathsf{a} \vee \neg \mathsf{b})) \wedge (\neg \mathsf{c} \vee \mathsf{d}) \wedge (\neg \mathsf{d} \vee (\neg \mathsf{a} \wedge \neg \mathsf{b}))$

Exercise 2 (10 Points, Models)

Consider $\mathcal{P}rop =_{def} \{\mathsf{r}, \mathsf{s}, \mathsf{t}\}$. How many SL valuations (models) over $\mathcal{P}rop$ are there for the following formulae? For each formula, state an equivalent formula in the language of SL (i.e. no macros allowed) that is as short as possible (i.e. the number of symbols (not counting parentheses) is as small as possible). We are omitting parentheses for better readability when irrelevant for the truthvalue of the formula.

(a) $\neg(\neg \mathsf{r} \rightarrow \neg \mathsf{r})$

(b) $\neg((\mathsf{r} \rightarrow \neg \mathsf{s}) \wedge (\mathsf{t} \rightarrow \neg \mathsf{s}))$

(c) $\mathsf{r} \leftrightarrow \mathsf{t} \leftrightarrow \mathsf{r}$

(d) $\neg(\square \wedge ((\mathsf{r} \wedge \mathsf{s}) \rightarrow (\mathsf{s} \vee \mathsf{t})))$

(e) $\mathsf{r} \wedge (\neg(\neg \mathsf{s} \wedge \mathsf{t}) \rightarrow \neg(\neg \mathsf{s} \rightarrow \mathsf{t}))$

(f) $\neg \mathsf{r} \rightarrow (\neg \mathsf{s} \rightarrow \mathsf{t})$

To be submitted:

16. April 2019
10:00 via box $\vee$ email $\vee$ exercise class

Exercise 3 (0 Points, Extra (13 Points): Boolean connectives)

In the lecture, we have defined SL with only the two connectives $\neg$ and $\vee$. All other connectives $\wedge, \rightarrow, \leftrightarrow$ were defined as macros. We mentioned, that we could have also used $\neg$ and $\wedge$ as the only connectives.
In this exercise, we consider the question: Can we build SL on *just one single binary Boolean connective* $\uparrow$? So we define $\mathsf{Fml}^{SL}_{\mathcal{L}(\mathcal{P}rop)}$ (Definition 2.2) as follows

$$\varphi ::= \square \mid \mathsf{p} \mid (\varphi \uparrow \varphi)$$

where $\mathsf{p} \in \mathcal{P}rop$.

(a) How many semantics for a binary connective $\uparrow$ do exist?

(b) How do they look like for these versions of SL? Write them down explicitly in the style of Definition 2.5 of the lecture.

(c) There are exactly two semantics, denoted by $\uparrow_1$ and $\uparrow_2$, such that all other boolean connectives ($\neg, \rightarrow, \leftrightarrow, \vee, \wedge$) can be defined as macros. Find them.

(d) Find a semantics for $\uparrow$ where $\neg$ can **not** be expressed and prove that it can't.

(e) Are the inference rules $\frac{\alpha, \alpha\uparrow_i(\beta\uparrow_i\gamma)}{\gamma}$, and $\frac{\alpha, \alpha\uparrow_i\beta}{\beta}$ correct inference rules in a calculus for $\uparrow_1$ or $\uparrow_2$? Prove or disprove.

(f) Consider any formula in $\mathsf{Fml}^{SL}_{\mathcal{L}(\mathcal{P}rop)}$ that uses only one propositional constant, i.e. $(a \uparrow_1 a) \uparrow_1 (a \uparrow_1 (a \uparrow_1 a))$. Find a simple algorithm to determine whether such a formula is a tautology and prove its correctness.